Analytical and numerical calculus of the elements of a transmission system

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The project consists in designing, modeling but also checking the elements of a 5-speed transmission system. During this work I set out to make a detailed analysis of the essential principles underlying the design, sizing, verification of strength and stiffness in the case of circular section shafts, used in power transmission (transmission system), the constitutive elements of such a system consisting of straight shafts and gears. The paper contains several ways to identify and verify the values resulting from the calculation in software applications such as MD Solids, but also a finite element analysis performed using the Ansys software.

1. Introduction

This study aims to perform a detailed analysis of the essential principles underlying the design, sizing, strength and stiffness testing in the case of circular section shafts, used in power transmission (transmission system), presentation of components for a transmission system, composition such as a system consisting of straight shafts and gears, the analytical and numerical calculation of the constituent elements.

2. Design of components [3], [4], [5]

The constitutive elements of a transmission system are: the protection shield, the input shaft, the intermediate shaft, the output shaft and the gears of different diameters.

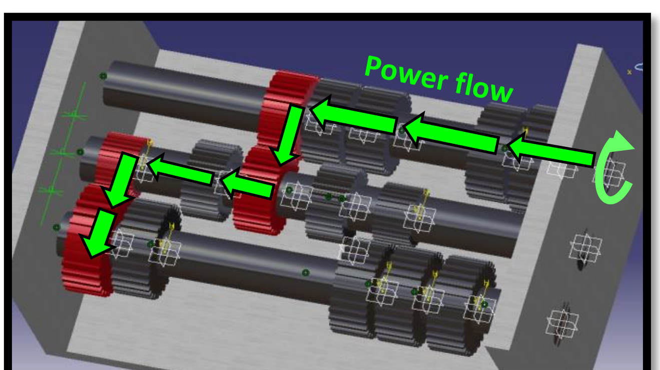

Fig. 1. Set of organological components

After designing and modeling the component elements, the assembly is performed to obtain the final structure of the transmission system, following the analytical and numerical calculation of the elements in the system. For example, the stress that occurs in shafts with a diameter of 40 mm is produced by a torque, and the maximum stress in the system is identified on the output shaft of the first gear (shaft 3).

In Fig. 1 it is shown the concept of the transmission system developed in the CATIA software. Also in this figure it was represented the concept of transmitting the power flow generated by the engine on the first gear.

In addition to the above, one can observe details of modeling and design of gears.

As a prelude to the 2D design and transformation (modeling) of three-dimensional gears, it is necessary to pay more attention to the elements defining the main characteristics of these organological components.

Regarding the main characteristics of the gears, some of them are listed below:

- foot circle;
- dividing circle;
- end circle;
- tooth width;
- tooth height;
- tooth profile.

The creation of the 2D sketch starts from the selection of all the previous features.

Fig. 2 shows the dimensioning principle of a single spur gear with a splitting diameter of 60 mm. In this context, the dimensions for the dividing circle, the end circle, the module (distance between 2 homologous points) and the foot circle have been chosen. In the case of the other wheels, this principle presented in Fig. 2.

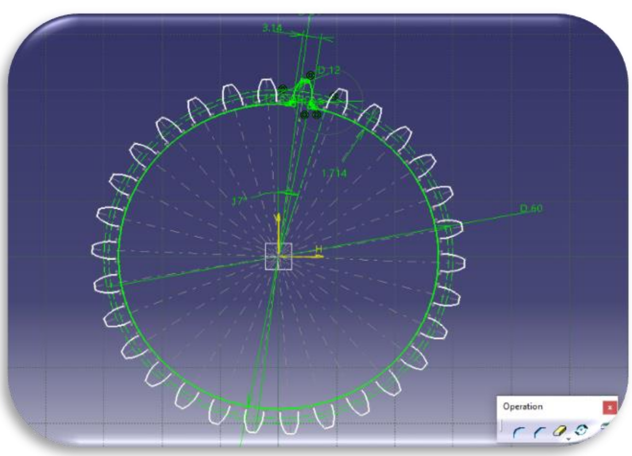

Fig. 2 Dimensioning of the gears.

Fig. 3. shows a solution in three-dimensional format of the gear with a diameter of division equal to 60 mm. To design the 3D model we start from the 2D model (basic model), and by simply using of the commands in CATIA, the complete shape of the gears is achieved. Each spur gear has its own 2D model, the common element occurring on any wheel is the module. This module does not differ, as it is possible to gear and stabilize the system during operation. The module represents the region of the dividing diameter that belongs to a tooth, the range of modules is established by STAS 822-82.

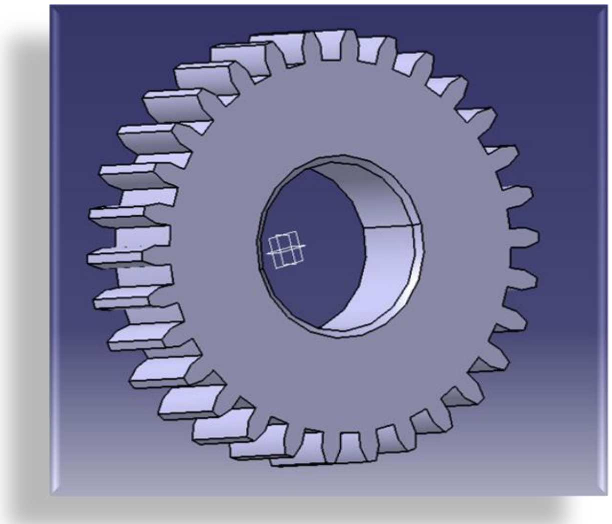

Fig. 3. Modeling wheels in 3D format

3. Analytical calculus of the shaft

In order to be able to perform analytical calculations for all gears, we start from a basic principle, so we made the schematic of the assembly using simple geometric figures

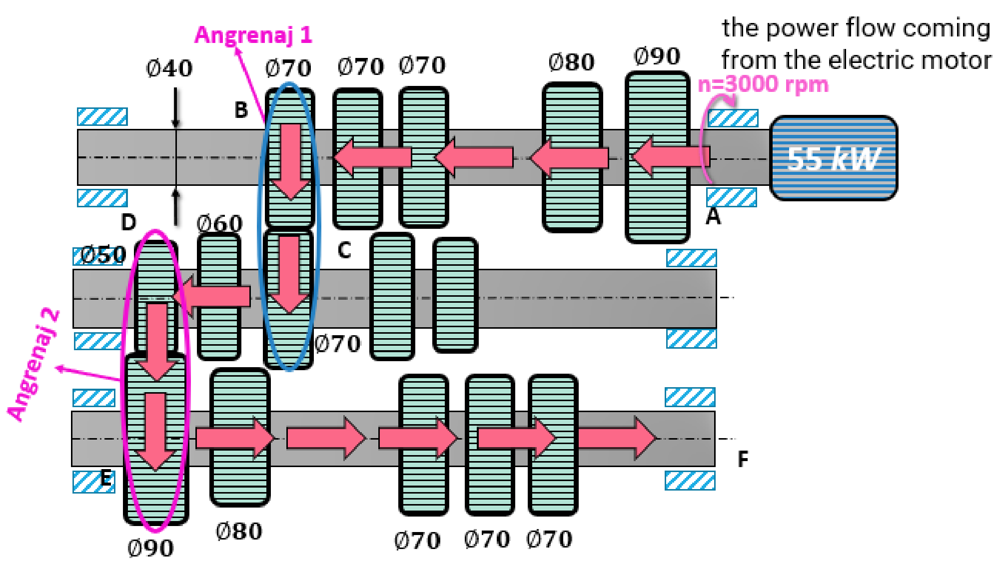

Fig. 4. Schematization of the assembly

Having known: the engine power (P = 55 kW) and the speed where the maximum torque is generated (n = 3000 rpm), it is possible to calculate the torque on the input shaft.

$$M_{tAB} = \frac{30 \cdot P}{n \cdot \pi} = \frac{30 \cdot 55}{3000 \cdot \pi} = 0.1750704\ kNm = 175.0704\ Nm\ (1)$$

$$M_{tAB} = 175.0704\ Nm\ (2)$$

$$M_{tAB} = F_B \cdot 35 \Rightarrow F_B = \frac{M_{tAB}}{35} = \frac{175070.4}{35} = 5002.011428\ N\ (3)$$

$$F_B = 5002.011428\ N\ (4)$$

$$M_{tD} = F_B \cdot 35 = 5002.011428 \cdot 35 = 175.0704\ Nm\ (5)$$

$$M_{tD} = 175.0704\ Nm\ (6)$$

$$F_D \cdot 25 = M_{tCD} \Rightarrow F_D = \frac{M_{tCD}}{25} = \frac{175070.4}{25} = 7002.816\ N\ (7)$$

$$F_D = 7002.816\ N\ (8)$$

$$M_{tEF} = F_D \cdot 45 = 7002.816 \cdot 45 = 315126.72\ Nm\ (9)$$

$$M_{tEF} = 315126.72\ Nm\ (10)$$

$$\tau_{AB} = \frac{M_{tAB}}{W_p}; \text{where } W_p \text{ is polar modulus (11)}$$

$$W_p = \frac{\pi \cdot d^3}{16}; W_p = W_{p2} = W_{p3}\ (12)$$

$$\tau_{AB} = \frac{175070.4}{12566.37} = 13.93\ MPa \leq \tau_{a(S235)}\ (13)$$

⇒ the shaft resists to torsional stress

$$\tau_{CD} = \frac{175070.4}{12566.37} = 13.93\ MPa \leq \tau_{a(S235)}(14)$$

⇒ the shaft resists to torsional stress

$$\tau_{EF} = \frac{315126.72}{12566.37} = 25.08\ MPa \leq \tau_{a(S235)}(15)$$

⇒ the shaft resists to torsional stress

What we can observe from the analytical calculation in the case of the first gear, is that the torsional stress does not produce destructive effects on the shafts. These organological elements (input, intermediate and output) withstand to torsional stress.

The same relationships and calculation steps are followed to calculate all gears.

4. Transient analysis [1]

4.1 Performing discretization in the case of a gear consisting of two wheels with identical diameters

Discretization is the transition from a continuous structure (with an infinite number of points) to a discrete structure (with a finite number of points) as one can see in Fig. 5.

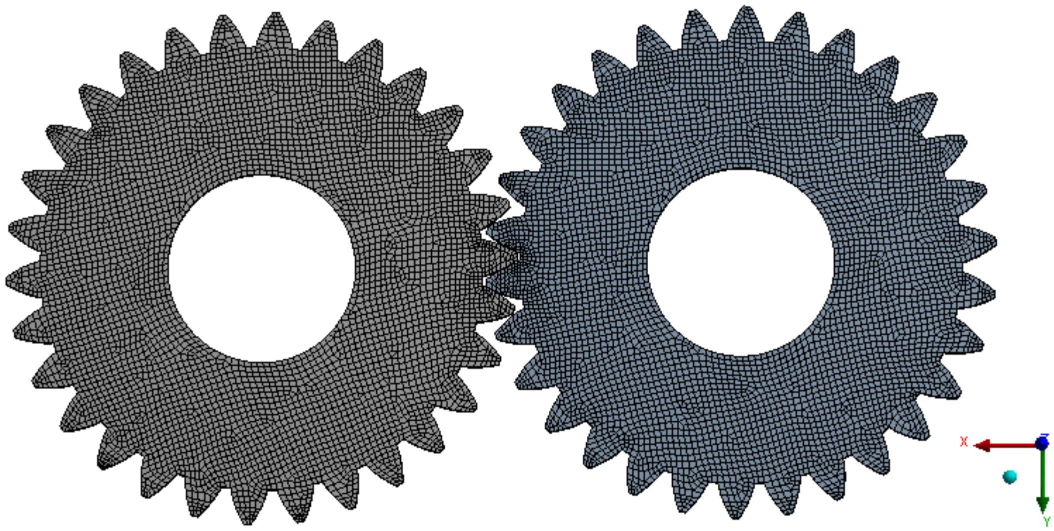

Fig. 5.Discretization

5. The convergence criterion of force

The aim of this analysis is to check if the force that appears in the gear (depending on the iteration) passes under the criterion (represented by the color blue)

This convergence graph is based on the NEWTON-RAPHSON method.

In the graph below, we notice the appearance of several "bisections", these bisections do not affect the analysis in this situation, because at half the time (for the next iteration) the force returns below the convergence criterion. The bisection occurs when there is an imbalance in the system, ie the sum of the forces formed is not equal to the sum of the reactions.

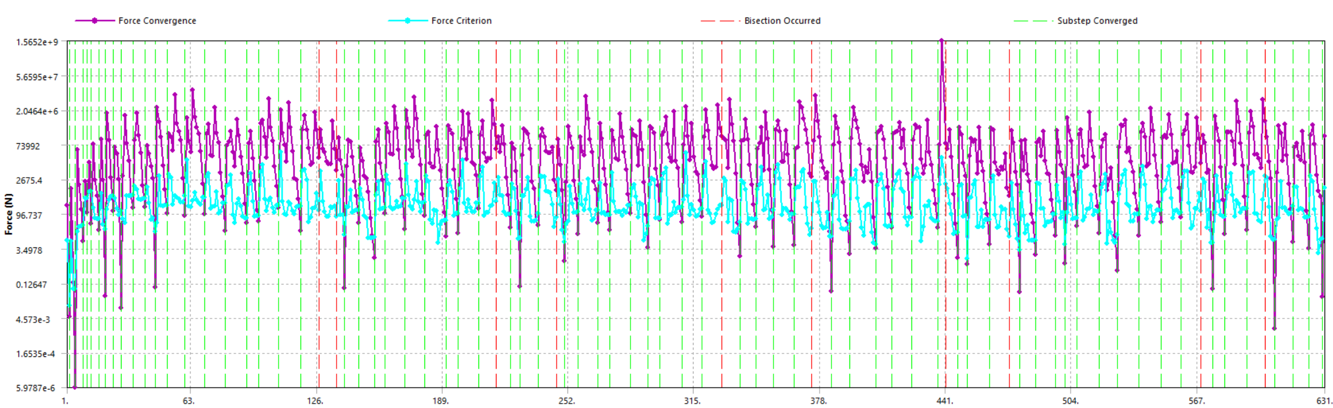

Fig. 6. Graph of convergence of forces

6. Interpretation of the analysis

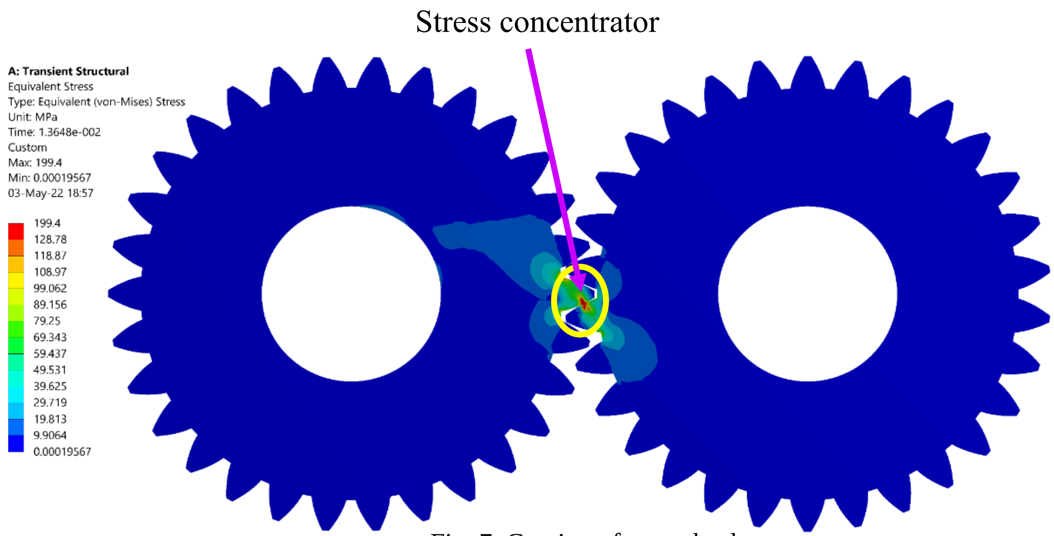

Fig. 7. Gearing of two wheels

Following the experiments performed on the gear in Fig. 7, the maximum value found was 199.4 MPa, in addition to this in the set of output data that the program provides we can find the values for each iteration, these output data can be seen in the graph below.

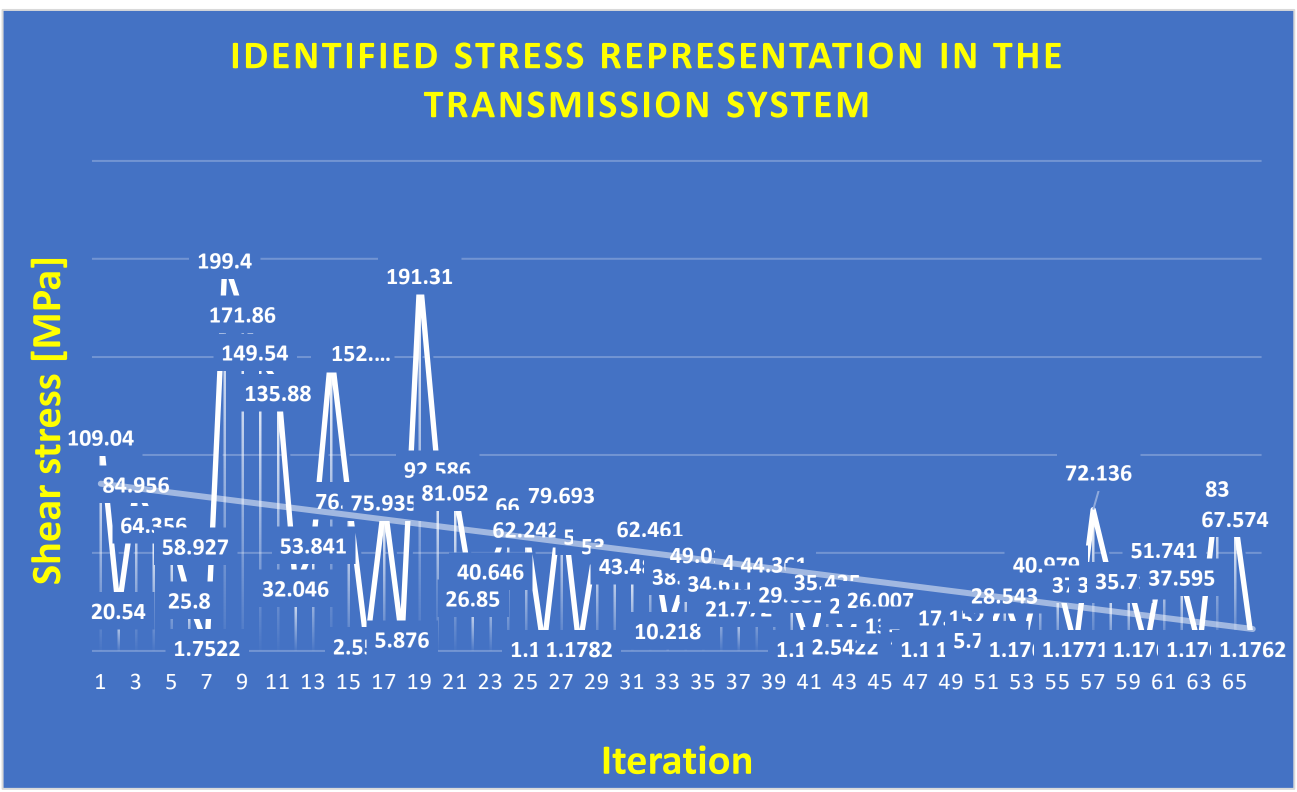

Fig. 8. Shear stress variation

7. Interpretation of output data

The table above is based on the value of the stress on the y-axis, and on the x-axis the time (iteration) where the respective stress is calculated. These values are displayed by the program at the end of the analysis, which can then be processed in EXCEL.

Iterație	Pași de timp	Tensiuni
1	1.00E-04	109.04
2	2.00E-04	20.54
3	3.50E-04	84.956
4	1.42E-03	64.356
5	1.93E-03	58.927
6	3.82E-03	25.8
7	9.80E-03	1.7522
8	1.36E-02	199.4
9	1.94E-02	171.86
10	2.52E-02	149.54
11	3.09E-02	135.88
12	3.67E-02	32.046
13	4.25E-02	53.841
14	5.11E-02	152.86
15	5.98E-02	76.631
16	6.19E-02	2.5503
17	6.41E-02	75.935
18	6.63E-02	5.876
19	6.95E-02	191.31
20	7.44E-02	92.586
21	7.92E-02	81.052
22	8.41E-02	26.85
23	8.90E-02	40.646

24	9.63E-02	66.262
25	9.99E-02	62.242
26	0.10357	1.1804
27	0.10904	79.693
28	0.11178	1.1782
29	0.11452	58.532
30	0.11725	53.752
31	0.11999	43.484
32	0.1241	62.461
33	0.1282	10.218
34	0.13436	38.114
35	0.1436	49.015
36	0.15283	34.611
37	0.15745	21.772
38	0.16207	45.624
39	0.169	44.361
40	0.179	29.032
41	0.184	1.1766
42	0.189	35.425
43	0.1965	2.5422
44	0.204	23.322
45	0.214	26.007
46	0.224	13.142
47	0.2275	7.654

47	0.2275	7.654
48	0.231	1.1761
49	0.23625	17.152
50	0.23887	1.1761
51	0.2415	5.7936
52	0.24412	28.543
53	0.24675	1.1767
54	0.25068	40.979
55	0.25659	37.26
56	0.2625	1.1771
57	0.27136	72.136
58	0.28022	34.135
59	0.28907	35.714
60	0.2935	1.1763
61	0.29793	51.741
62	0.30458	37.595
63	0.3079	1.1761
64	0.31122	83
65	0.31455	67.574
66	0.31953	1.1762

Fig. 9.Centralization of values

8. Equivalent Elastic Strain

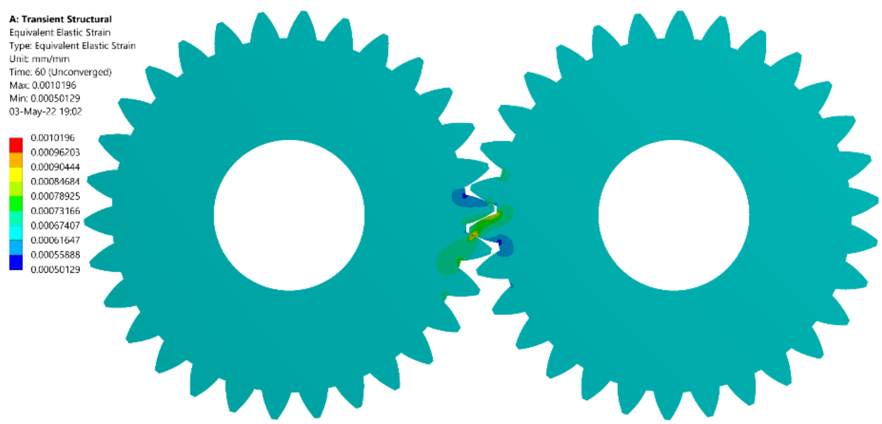

Fig.10 Equivalent elastic strain occured at the teeth contact

From the analysis regarding the strain, we notice that the maximum value is 0.00196 $\frac{mm}{mm}$

9. Shear Stress

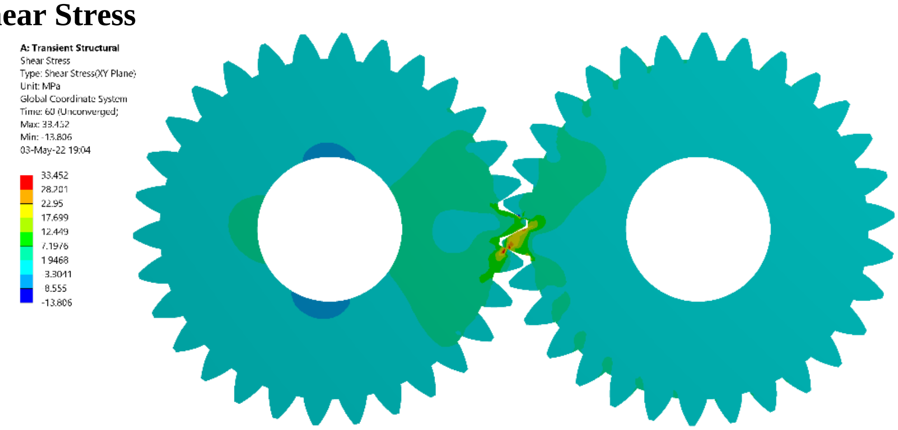

Fig.11 Shear stress occurred at the teeth contact

10. Conclusions

Following the interpretation of the analysis in transient mode, one can argue that the assembly consisting of two gears of identical diameter withstands the stresses occured in the system, the maximum stress at the iteration level (over time) is less than the maximum allowable stress of steel ($\tau_{all} = 220$ MPa).

Due to the numerous tests performed on the principles presented above at the level of gears, using the finite element method (FEM), even if in other gear variants stresses of thousands of MPa could appear (at a few iterations, ex 1000 MPa - 1500 MPa) we cannot draw wrong conclusions, since these local stresses, due to the type of contact between the two gears produce a small pattern contact (Hertzian contact).

Values between 1000 MPa and 1500 MPa are normal because the gears are designed to withstand with such stresses on the sidewalls.

Based on the above, the final conclusion is that the kinematic and dynamic operating conditions of the transmission system are observed, but especially the strength conditions.

11. Bibliography

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