LINEAR BUCKLING ANALYSIS OF STIFFENED PANELS

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This article investigates the buckling behaviour of stiffened panels commonly used in aeronautical engineering. Stiffened panels play a critical role in aircraft structures, providing structural integrity and load-bearing capabilities. The article outlines the analytical approach based on classical plate theory and a finite element method (FEM) analysis of a simply supported thin plate. This example highlights the applicability of both approaches and emphasizes the role of numerical simulations in complex structural analyses. Moving beyond the simple plate example, the article then focuses on the linear buckling analysis of a stiffened panel that resembles a real aircraft structure. The panel's geometry and boundary conditions are defined to simulate the actual operating conditions of the aircraft. Furthermore, a comparative analysis is conducted to evaluate the buckling performance of a similar panel without stiffeners. This comparison highlights the essential role of stiffeners in designing aircraft structures.

KEYWORDS: linear buckling, stiffened panels, aeronautical engineering.

1. Introduction

For any given loading condition of the aircraft the structure is stressed in tension, shear, or compression. In tension, ultimate load is usually limited by the quality of the material available. However, in case of compression, thin-walled structures are, by definition, susceptible to loss of elastic stability (buckling). In most cases, ultimate strengths of stiffened panels are primarily dependent on the stability behaviour, therefore stiffened panels are widely used, due to their high strength-to-weight ratio and ability to withstand complex loading conditions.

According to [1], the components of stiffened panel may be classified as follows:

- Longitudinal reinforcing members- stringers and spars, that can carry appreciable tensile loads and, when supported, compressive loads as well.
- Skin- tensile, compressive and shear loads can be carried, but reinforcement (lateral support) is required. The thin skin used in aircraft structures can only sustain and transmit normal pressure over short distances by bending.
- Transverse reinforcing members- ribs, that provide in-plane stiffness and strength, being incapable of carrying much lateral load.

A typical wing section is presented in Fig. 1:

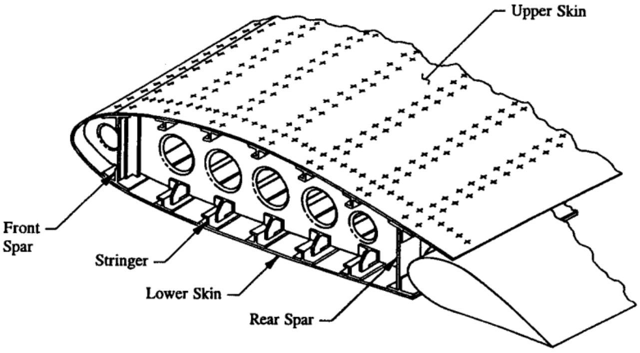


Fig. 1 Main wing components [2].

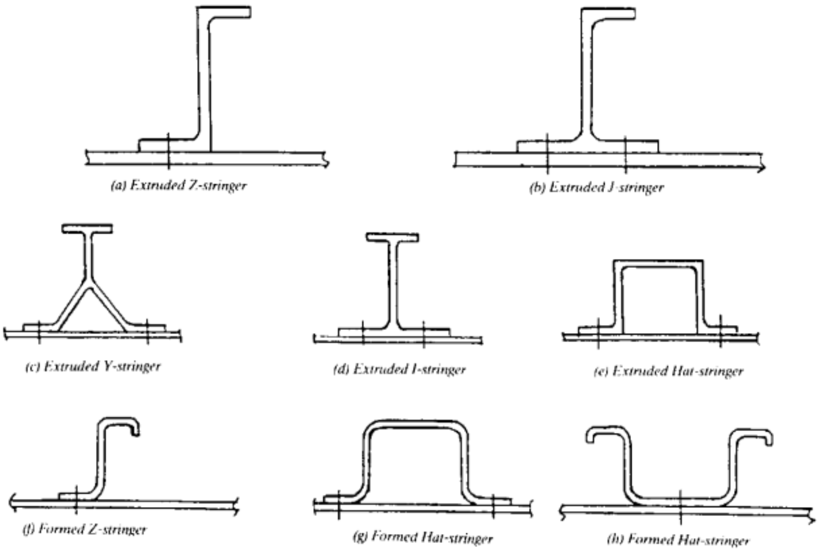
The key initial instability modes of stiffened panels are:

- Skin Local Buckling- generally involves waving of the skin between stringers in a half-wavelength comparable with the stringer pitch [5].
- Stiffener Local Buckling- out-of-plane stringer web or flange displacement in a half-wavelength comparable with the stringer depth [5].
- Inter-rivet Buckling- buckling of the skin as a short strut between rivets [5].
- Panel Global Buckling- buckling of a panel as a whole, often a catastrophic stiffness reduction that can lead to structural collapse.

After the initial knuckling, following failure can occur:

- Stiffener Column Buckling- once buckled, skin is no longer able to carry the applied loads, resulting in a column-like behaviour of stiffeners [1].
- Crippling Failure- local distortion of the cross-sectional shape, the beginning of which usually occurs at the load appreciably less than the failing load with more stable portions of the cross-section continuing to take additional load while supporting the already buckled portion until complete collapse occur [1].

A key aspect of stiffened panel stability is stringer's section. There is a multitude of different cross-section of stiffeners used in aircraft design, the typical skin-stringer configurations are shown in Fig. 2:



(a) Extruded Z-stringer

(b) Extruded J-stringer

(c) Extruded Y-stringer

(d) Extruded I-stringer

(e) Extruded Hat-stringer

(f) Formed Z-stringer

(g) Formed Hat-stringer

(h) Formed Hat-stringer

Fig. 2 Typical skin-stringer configurations [1].

Z-stiffened panels are efficient structural elements for compressive load. Other important merits of Z-stiffened panels include ease of assembly and good accessibility for inspection [6]. Although panels with closed section stiffeners, such as hat-shaped stiffeners, are slightly more efficient than Z-stiffened panels, they are more difficult to inspect.

Other than the stringer type, a skin-stringer configuration is defined by width and thickness of the stiffeners and the stiffener spacing. Following dimensions were considered for this study: frame spacing of 600 mm (rib pitch) and stiffener spacing of 250-300 mm (stringer pitch).

2. Finite Element Models

This study comprises three distinct finite element models (FEM), each designed to investigate a specific aspect of buckling analysis for thin panels. These models include:

- An analytical relation-based example of a thin rectangular plate with all edges simply supported, intended to validate the accuracy and reliability of buckling analysis techniques for thin panels.
- A FEM of a thin stiffened plate with dimensions closely resembling those of a real aircraft structure, specifically the horizontal stabilizer of a private jet.
- A FEM of a thin plate with identical dimensions to the stiffened plate. The purpose of this model is to enable a comparison of the buckling behaviour of the plate with and without stringers, highlighting the effects of stringers on the buckling behaviour of aircraft structures.

The general buckling equation for a thin rectangular plate in compression [3] (see Fig. 3) allows to determinate the critical buckling stress:

$$\sigma_{cr} = \frac{k\pi^2 E t^2}{12(1-\mu^2)b^2} \tag{1}$$

Using relation (1), the critical buckling load for a plate with following dimensions was determined: a=300 mm, b=100 mm, t=1 mm, μ=0.33, E=71000 MPa, k=4 (for all edges simply supported).

$$\sigma_{cr} = \frac{k\pi^2 E t^2}{12(1-\mu^2)b^2} = \frac{4\cdot\pi^2\cdot 71000\cdot 1^2}{12(1-0.33^2)100^2} = 26.21\ \text{MPa} \tag{2}$$

A thin plate lengthened in the direction of loading, with an aspect ratio of three, will buckle into three waves, each of them being square and acting as shown in Fig. 3 (at the left):

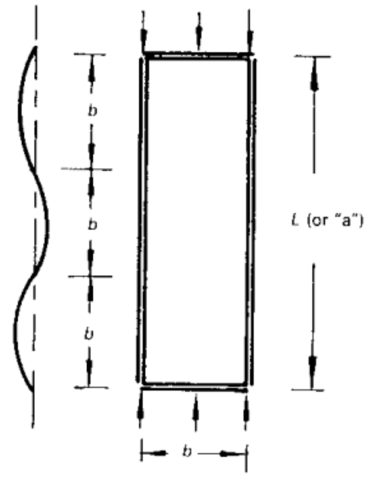


Fig. 3 A rectangular plate with all edges simply supported [1].

From the load determined with analytical relation (1) the critical buckling force was determined:

$$F_{cr} = \sigma_{cr}\cdot a\cdot t = 26.21\cdot 100\cdot 1 = 2621\ \text{N} \tag{3}$$

Considering given geometry, material and supporting conditions, a simple FEM was developed in ANSYS Workbench, student version (2022 R2) [4]. A schematic representation of the boundary conditions is presented in Fig. 4:

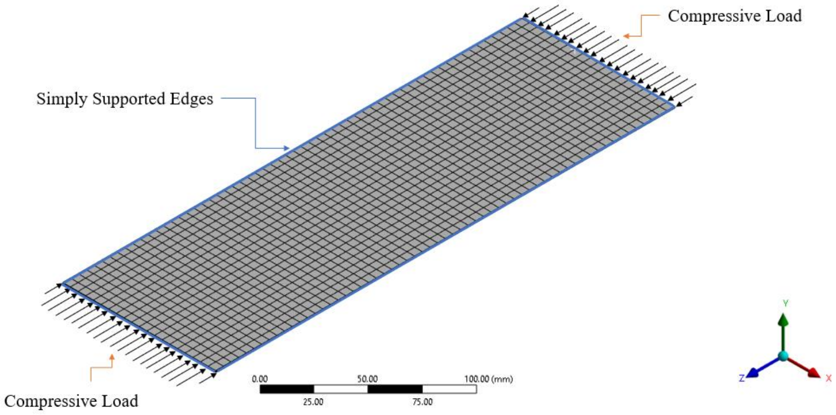


Fig. 4 FE model of a simply supported thin plate loaded in compression.

For the second part of the study, a model of a thin stiffened panel with dimensions and supports condition resembling real aircraft structure was analysed. As a reference, a private jet plane horizontal stabilizer (HS) was considered. A simplified cross section of the lower HS skin is presented in Fig. 5:

Fig. 5 Sequence of stiffeners along HS skin panel.

A part of the skin panel was modeled, consisting of a sequence of four Z-stiffeners bonded to the skin. The general dimensions are presented in Fig. 6:

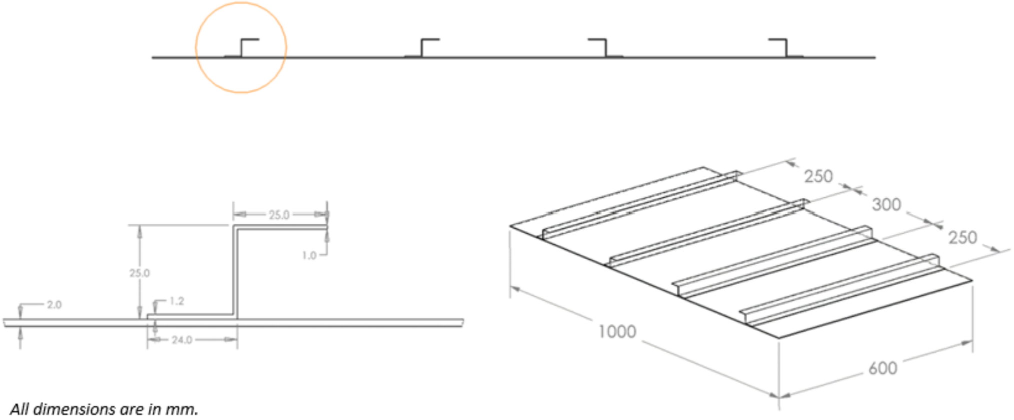


Fig. 6 Stiffened panel general dimensions. All dimensions are in mm.

Skin panels are supported in the transverse direction by ribs, providing a simply supported condition. Additionally, as only a part of the skin panel was analysed, a symmetry condition is applied to edges parallel to stringers. To determine the critical force value, a compressive load was applied, aiming for a resultant load multiplier factor close to one for the first buckling mode. The schematic representation of the boundary conditions is shown in Fig. 7. For accurate analysis, a fine mesh was generated with an element size of 10 mm. The contact region was treated as bonded and modelled using CONTA174 and TARGE170 elements.

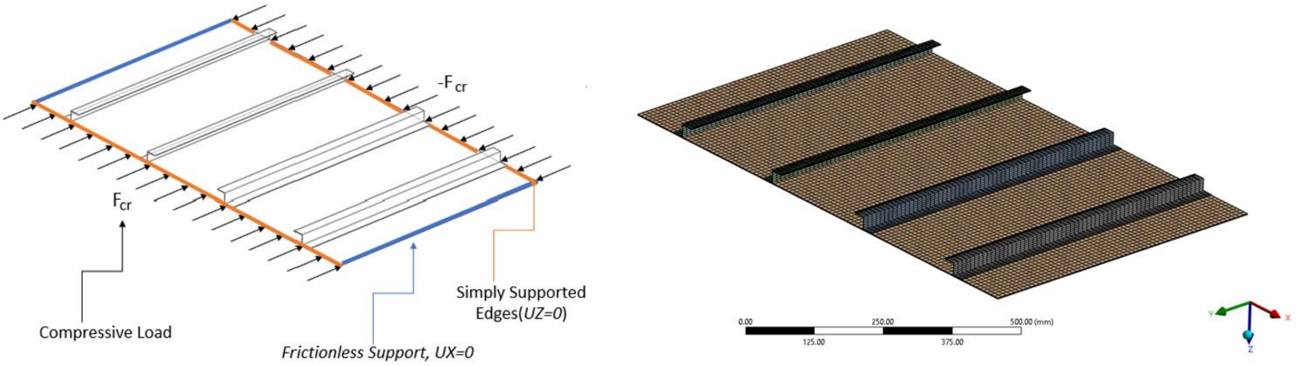


Fig. 7 Boundary conditions and the mesh of stiffened panel model.

Additionally, to enable a comparison of the buckling behaviour of the plate with and without stringers, a third FEM of a thin plate with identical to the stiffened panel dimensions and boundary conditions was analysed.

3. Results and Discussions

Using the Eigenvalue Buckling module [4], the first buckling mode of the simply supported thin plate was determined. The obtained load multiplication factor was approximately one (1.03), indicating that the applied load can be considered critical. As described in [1] the thin plate under investigation was observed to buckle into three waves, as shown in Fig. 8. This result confirms the validity of the model and demonstrates its capability to accurately capture the buckling behaviour of thin plates.

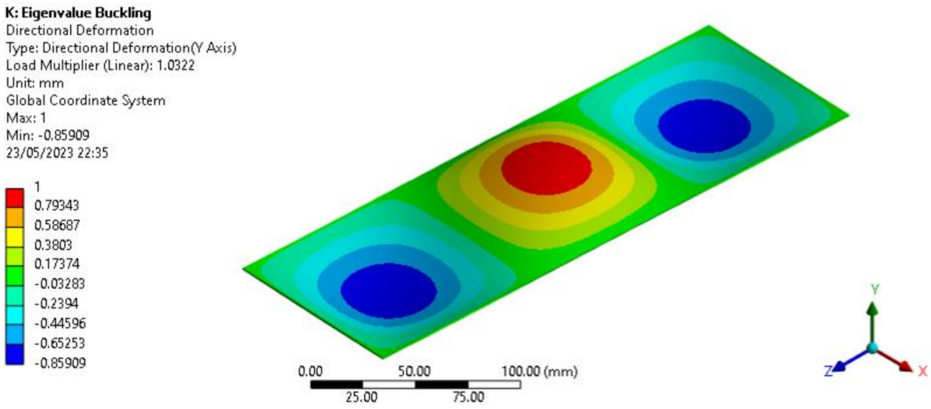


Fig. 8 The first buckling mode of the rectangular simply supported thin plate.

For the second example, the first four buckling modes corresponding to an axial compressive load of 25000 N were determined. For the first buckling mode a load multiplier factor of 1.07 was obtained. Results are presented in Fig. 9:

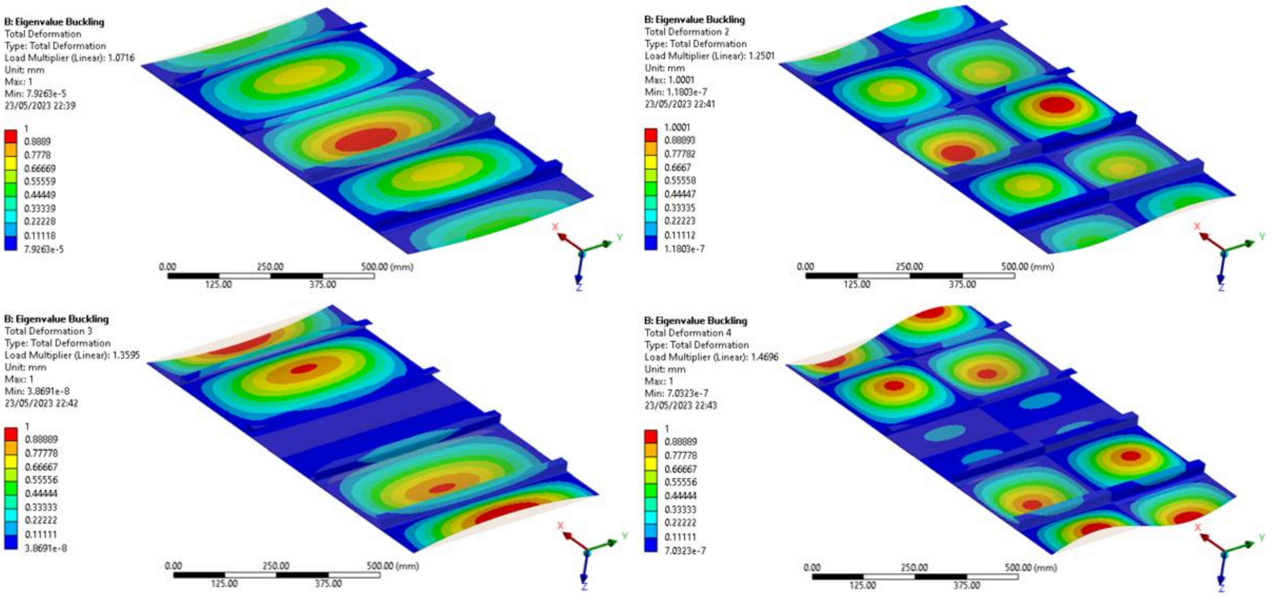


Fig. 9 The first 4 buckling modes of the stiffened panel.

The same approach of determining critical buckling force was applied for the analysis of panel without stiffeners. An axial compressive force of 1400 N was used to obtain the first four buckling modes. For the first one a load multiplier of 1.04 was obtained. Fig. 10 shows the first four buckling modes for panel without stiffeners:

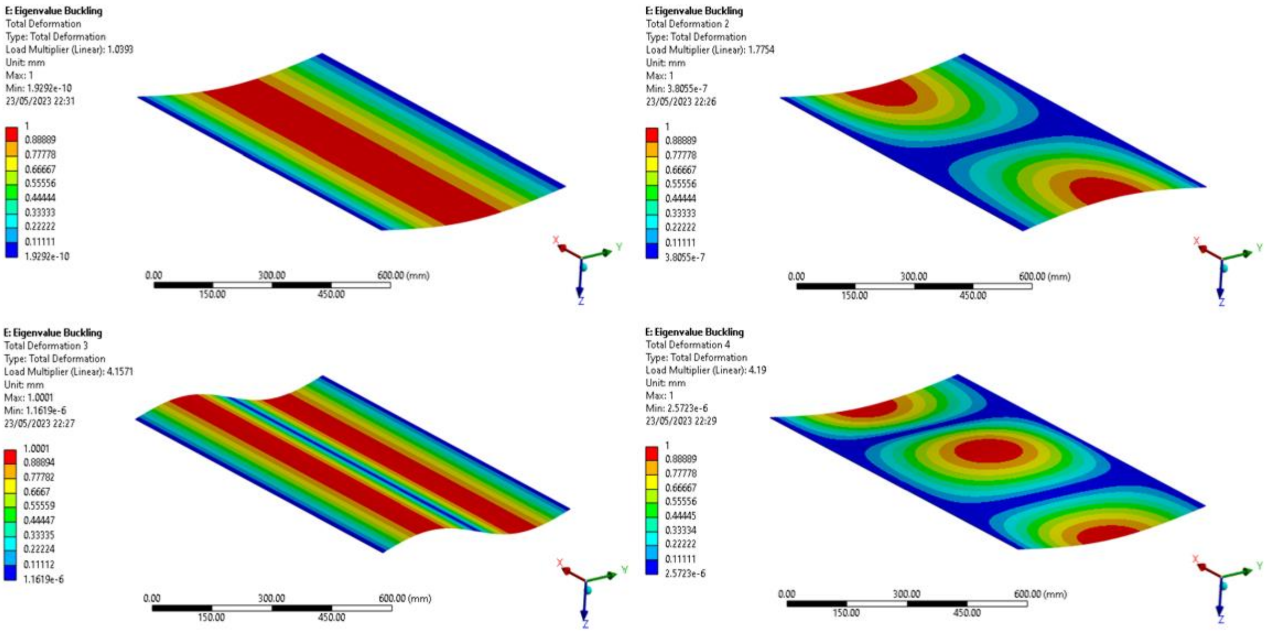


Fig. 10 The first 4 buckling modes of the panel without stiffeners.

The results derived from the analysis of both stiffened and unstiffened panels provide insight into the influence of stringers on the buckling behaviour of thin plates. Table 1 presents the critical buckling load for the two plate types. The presence of stiffeners increased F_{cr} by 23600 N, resulting in a 17.85 times greater critical buckling force. Such improvement is a clear example of stiffener's high strength-to-weight ratio and ability to withstand demanding loading conditions.

Table 1 Eigenvalue Buckling Results

Results	Panel with stiffeners	Panel without stiffeners
Critical Buckling Load [N]	25000	1400
Load Multiplier [-]	1.07	1.03

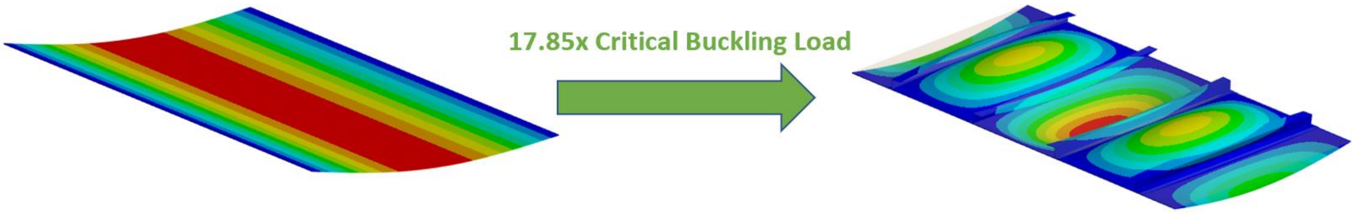


Fig. 11 Increase in critical buckling load: panel with and without stiffeners.

Moreover, the introduction of stiffeners alters the buckled shape of the plate. Fig. 9 illustrates how the skin locally buckles between stringers, enabling load redistribution within the skin-stringer configuration and creating a structure capable of carrying load even in a post-buckled state. On the other hand, for panels without stringers, the first mode of buckling resembles a column (Fig. 11). As the stability of the plate is greatly influenced by boundary conditions, a plate with 2 free edges will behave as a column once it reaches the critical buckling stress. This transition is presented in Fig. 12:

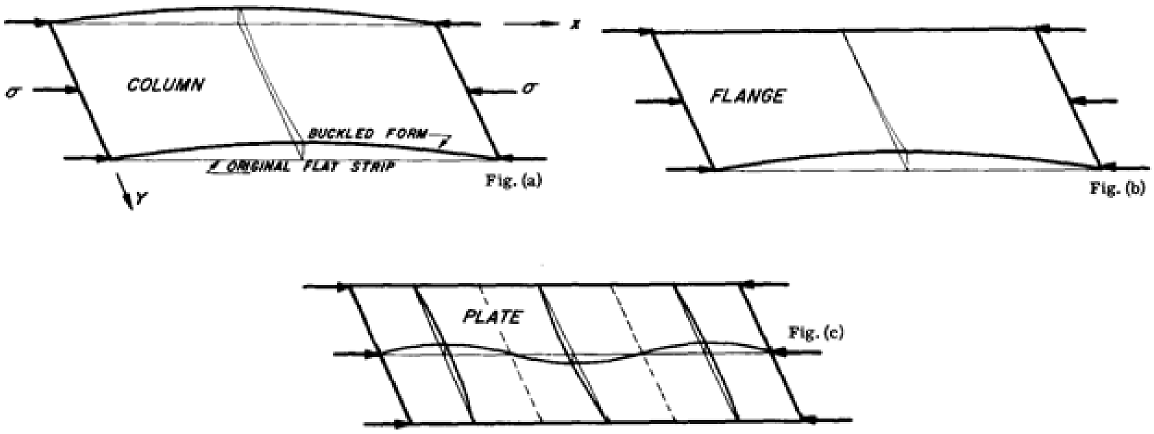


Fig. 12 Transition from column to plate as supports are added along unloaded edges [7\]

Initial buckling of a stiffened panel consists of a local buckling mode which is a mixture of skin buckling, local instability and torsional intability, the predominant type of buckling being dictated by the geometry of the particular skin-stringer configuration used [1]. Understanding the role of these factors are extremely important for designing and analyzing stiffened panels, especially in aerospace applications, where weight and structural performance are crucial.

4. Conclusions

Three distinct finite element models were built to investigate various aspects of buckling analysis for thin panels. Each model served a specific purpose and highlighted the essential role of stiffeners in designing aircraft structures.

By comparing the available analytical results with FEM results for a simple geometry, with distinct boundary conditions, the study confirmed the effectiveness of the chosen analysis techniques in predicting buckling behaviour accurately.

A side-by-side comparison of a panel with and without stiffeners revealed a significant increase in load-carrying capabilities when stiffeners were present. As in the aeronautical field weight is a crucial parameter, lightweight structures with a high strength-to-weight ratio are of great interest. By considering the specific geometry of the skin-stringer configuration, engineers can predict and address the primary mode of buckling more accurately, enabling them to create the most optim designs.

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Nomenclature

FEM: Finite Element Model
σ_{cr}: critical buckling stress
F_{cr}: critical buckling load
k: non-dimensional constants that depend upon condition of edge restraint and shape of the plate
E: Young modulus
t: thickness of the plate
b: width of the plate
a: height of the plate
μ- Poisson’s ratio
HS- Horizontal Stabilizer